P.R.GOVT.COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - Semester V (w.e.f. 2018-2019)
Course: Ring Theory & Vector Calculus

Total Hrs. of Teaching-Learning: 45 @ 3 hr/Week **Total credits: 3**

Objectives:

- To impart knowledge on Ring Theory and its applications.
- To make awareness of the concepts of the transformation between line Integral, Surface Integral and Volume integral.
- To introduce the concepts of geometrical meaning of Gradient, Divergence and Curl.

Unit – I: Rings – I **(11 hrs)**

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws in Rings, Integral Domain, Division Ring and Fields, The characteristic of a ring – The characteristic of an Integral Domain, the characteristic of a Field, Sub rings and Ideals.

Unit – II: Rings – II **(11 hrs)**

Definition of Homomorphism – Homomorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental Theorem of Homomorphism – Maximal Ideals – Prime Ideals.

VECTOR CALCULUS

UNIT:III.Vector differentiation **(9 hrs)**

Vector differentiation –Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient , Divergence, Curl operators, Formulae involving these operators.

UNIT:IV.Vector integration: **(7 hrs)**

Line Integral, Surface Integral, Volume Integrals with examples.

Unit – V: Vector Integration Applications: **(7 hrs)**

Gauss Divergence Theorem, Stokes theorem, Green's Theorem in plane and applications of these theoremss.

Additional Inputs : Euclidean Ring definition and Examples.

Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

Books for Reference:

1. Topics in Algebra by I.N.Herstine
2. Abstract Algebra by J. Fralieh, Published by Narosa Publishing house
3. Vector Calculus by Santhi Narayan, Published by S.Chand & Company Pvt. Ltd., New Delhi
4. Vector Calculus by R.Gupta, Published by Laxmi Publications.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-V, PAPER V

Unit	TOPIC	V.S.A.Q	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Rings – I	02	03	01	25
II	Rings – II	02	02	O2	28
III	Vector differentiation	01	02	01	19
IV	Vector integration	01	02	01	19
V	Vector Integration Applications	02	01	01	15
TOTAL		08	10	06	106

E.Q = Essay questions (8 marks)
S.A.Q = Short answer questions (5 marks)
V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 4x8M = 32
Short answer questions : 5x6M =30
Very Short answer questions : 8x1M =08

Total Marks : 70

P.R.Government College (Autonomous), Kakinada
III year B.Sc., Degree Examinations V Semester
Mathematics: Ring Theory & Vector Calculus
Paper–V (Model Paper w. e. f. 2018-2019)

Time: 3 hours **Max. marks : 70M**

PART –I

Answer all the following questions. Each question carries 1 mark. 8x1M =8M

1. Define Boolean Ring.
2. Write the zero divisors of $(Z_9, +_9, X_9)$.
3. Find Kernel of the Homomorphism $f: Z(\sqrt{2}) \to Z(\sqrt{2})$ defined by $f(m+n\sqrt{2}) = m - n\sqrt{2} \ \forall\, m+n\sqrt{2} \in Z(\sqrt{2})$.
4. Give an example to show that every prime ideal need not be a maximal ideal.
5. Find div f, where $f = grad(x^3 + y^3 + z^3 - 3xyz)$
6. Evaluate $\int_0^1 (e^t \bar{i} + e^{-2t} \bar{j})dt$
7. State Green's theorem
8. State the Green's Identities.

PART –II

Answer any THREE questions from each section. Each question carries 5 marks. **6x5M =30M**

SECTION – A

9. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
10. Prove that the intersection of two ideals of a Ring R is an ideal of R.
11. Prove that a commutative ring R with unity having no proper ideals is a field.
12. Let R and R' be two rings and $f: R \to R'$ be a homomorphism. Then prove that the Kernel of f is an ideal of R.
13. Let C be the ring of Complex numbers and $M_2(R)$ be the ring of 2 x 2 matrices. If $f: C \to M_2(R)$ is defined by $f(a+ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ then prove that f is an into isomorphism and also find ker f.

SECTION – B

14. Find the directional derivative of $\phi = xy + yz + zx$ at A in the direction of $\overline{AB}$, Where A = (1,2,-1), B= (-1,2,3)
15. Prove that div Curl $\bar{f} = 0$
16. If $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, find the circulation of F round the curve, C where C is

the Circle $x^2 + y^2 = 1, z = 0$.

17. Evaluate $\int_V F dV$ when $F = x\bar{i} + y\bar{j} + z\bar{k}$ and V is the region bounded by x=0,y=0,y=6,z=4 and $z=x^2$.

18. Evaluate $\oint_C (\cos x \,.\, \sin y - xy)\, dx + \sin x . \cos y \, dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

PART –III

Answer any FOUR questions from the following by choosing at least ONE from each section. Each question carries 8 marks. 4X8M=32M

SECTION – C

19. Define the characteristic of a ring. Prove that the characteristic of an integral domain is either a prime or zero.
20. State and Prove fundamental theorem of homomorphism in rings.
21. Show that an ideal U of a commutative ring R with unity is maximal if and only if the quotient ring R/U is a field.

SECTION – D

22. Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A$

23. Evaluate $\int_S F.N\, dS$, where $F = zi + xj - 3y^2 zk$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

24. If $F = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ find $\int_S F.Nds$ by divergence theorem where S is surface of the cube bounded by x = 0,x=1 , y=0,y=1,z=0,z=1.

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